# The Entropy Level, Systematic Risk, and Market Sentiment[1]

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## Abstract

This paper attempts to measures the information level of the market via information entropy defined in Gosh et. al. (2016). The relation between the information level of the market, systematic risk, and market sentiment is investigated. The sentiment of the market is measured by the analyst forecasts of the market, where the systematic risk is measured by the dynamically estimated corporate beta. In general, there is a positive relation between the analyst forecast (proxy for sentiment) and corporate beta (proxy for systematic risk). In addition, the relation between the corporate beta (proxy for systematic risk) and entropy level of the market (proxy for market information) is presented.

Keywords: Kalman Filter, Time varying Beta, Information Entropy, Analyst Forecasts

## 1. Introduction

This paper attempts to identify the relation between the market information level defined by the market entropy and systematic risk as well as the market sentiment of the market.

There are several papers that attempt to capture the systematic risk of securities in a dynamic manner (e.g. Kalman filter estimation). There is empirical evidence that the dynamic behavior of systematic risk is better captured when it is estimated with the Kalman filter. Mamaysky, Spiegel and Zhang (2008) demonstrate the validity of the time varying beta model. Mamaysky et al. (2008)

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find that the Kalman filter fits the data better than other conventional models. Such findings support the conjecture that systematic risk is driven by linear projections of the future (just as the Kalman filter assumes). Under the assumption that investors behave as if they estimate the systematic risk using a Kalman filter, this paper aims to figure out how much information is extracted by implementing the Kalman filter.

Gsosh et. al. (2016) utilizes the minimization of the cross entropy of two probability distributions corresponding to the observed and risk neutral distribution that is known to be equivalent to constructing the distribution with the maximum likelihood. For a fixed entropy level, the deviation from the constraint, i.e. the fact that the expected return is zero under the risk neutral distribution, may be interpreted as the level of market incompletion.

The Kalman filter is implemented to dynamically estimate the market beta for listed firms traded in Japan which were used to obtain the time varying market beta. The systematic risk is estimated using high frequency data, i.e. tick level data, that is pooled at the 5 min. frequency after filtering out irrelevant quotes that seemingly did not lead to effective transactions. The Market sentiment is assessed using the average of the analyst forecast data of the overall market obtained from the QUICK database, which is compared to the systematic risk proxy (corporate beta), and the information entropy level (Kullback-Liebler entropy) of the market return distribution. The comparison reveals the sensitivity of systematic risk to market sentiment and market entropy level.

## 2. Model

This paper builds on the arguments of Takezawa (2022, 2021, 2023) in the sense, that the analysis is implemented on a higher frequency level of data, and verifies the usage of information entropy as an indicator of the market sentiment and systematic risk of the market. The motivation to use the Kalman filter and the information entropy level is reviewed from the arguments made in Takezawa (2022) in the following.

The calculation of the Kullback-Leibler (1951) information criterion was conducted for each market return distribution obtained on a daily basis. The probability distribution is estimated from a frequency distribution created from the historical return series of the market index (Nikkei 225) and the individual stock returns[2]. The entropy of the distribution was calculated by

$$I = -\sum_i p_i \log(p_i)$$

where $p_i$ is the relative frequency of the distribution at 20 fixed points. The calculations are summarized in Section 2.

The entropy is calculated based on the historical distribution and the forecasted distribution, respectively. If put in the relative entropy context, the entropy considered in this paper is the relative entropy from a constant (i.e. uniform distribution or uninformative distribution)[3].

The reduction in information entropy implies the potential use of the time varying Beta model. On the other hand, the presence of a stable level of information entropy supports use of the static Beta model. This means that corporate beta possess a higher level of stationarity. Because the error term is Gaussian and the entropy measure is invariant to the mean, one may consider that the analysis may be applied with the variance of the Gaussian distribution. Although the histograms are generated by the kernel smoothing technique, unless the error distribution is stationary, any deviation from the Gaussian distribution means using a time varying beta model is appropriate that supports the usage of an information entropy measure. Thus, it is meaningful to consider the entropy measure as a measure of information that may be extracted from the historical return distributions.

This market entropy depends on the information entropy level of the historical return distribution. The historical returns are "what we know" and by dynamically conditioning on this past information, the Kalman filter forecasts the future to estimate the market beta.

If the entropy level is low, this can be interpreted as if the past returns reveal sharper future forecasts. Subsequently, this implies a reduction in the future return distribution uncertainty, which is defined as a reduction in systematic risk and/or market sentiment in this paper. Betting against the change in market

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2 The distributions were obtained by applying kernel smoothing with a normal kernel function (nonparametric).

3 One may consider the relative cross entropy of the historical and forecasted distribution, which may also serve as a proxy for the information change.

sentiment may lead to trading profits, but this not the focus of this paper.

The Kalman filter introduced by Kalman (1960) is used to estimate the corporate beta[4] that serves as a proxy for systematic risk. The Kalman filter model used, has an AR(1) structure that is used to estimate the dynamic betas as the following:

$$S_t^i = c^i + \beta_t^i M_t^i + \varepsilon_t^i$$
$$\beta_{t+1}^i = a^i + \beta_t^i b^i + v_t^i$$

where $S_t^i$ is the stock return, $M_t^i$ is the market index return, $\beta_t^i$ is the time varying (dynamic) beta, $a^i$, $b^i$, $c^i$ are constants and $\varepsilon_t^i$, $v_t^i$ are uncorrelated disturbance terms. The superscript $i$ corresponds to the $i$ th firm.

### Kalman Filter Beta (long term)

The long-term persistent level of the dynamically estimated beta can be obtained by the following equation. Given that the auto regressive term is stable, i.e. $b^i$ is between 0 and 1.

$$\beta_{LT}^i = \frac{a^i}{(1-b^i)}$$

The advantage of this approach is that it is easier to extract stable time-varying betas when the underlying high frequency data contains a high level of noise. The attempt to use the long-term beta explores the power of incorporating "forward looking measures" implied from the historical data, which may serve as a proxy for the "expected return" of the market. The intraday returns are later averaged over the day to match the frequency of the long-term betas. The average long run beta is used to capture the future average beta (considering the limit time when sent to infinity).

The objective here is to observe the dynamic behavior of the intraday beta (long-term beta), and how it relates to the analyst forecasts on systematic risk as well as the information entropy level of the market. This analysis intends to reveal the systematic dynamics of the market using high frequency level data. The deviation between the systematic risk change and the analyst forecast level appear to have some relation, and this relation is revealed empirically using the following

4 Among the Kalman Filter estimates, only those who had a value between 0 and 10 were extracted.

model whose results are reported in section 3.

$$Y^i = c^i + \alpha^i X^i + v^i$$

where $Y^i$ is the dependent variable representing the difference between the analyst forecast level and realized level of the market (proxied by TOPIX), $X^i$ is the dependent variable representing the average of the long-term betas calculated by the Kalman filter. The index $i$ stands for the group of all of the firms, and the group of firms with their security code in the 1000s, 2000s, 3000s, 4000s, 5000s, 6000s, 7000s, 8000s, 9000s respectively. $c^i$ and $\alpha^i$ are the coefficients to be estimated in the regression model with Gaussian noise $v^i$.

A similar analysis is performed to measure the effectiveness of the information criteria constructed from the market information entropy level measured by the systematic risk proxy (long-term beta).

$$Z^i = c^i + \alpha^i X^i + \varepsilon^i$$

where $Z^i$ is the dependent variable representing the market information criterion defined by the Kullback-Leibler (1951) information criterion, $X^i$ is the dependent variable representing the average of the long-term betas calculated by the Kalman filter. The index $i$ stands for the group of all of the firms, and the group of firms with their security code in the 1000s, 2000s, 3000s, 4000s, 5000s, 6000s, 7000s, 8000s, 9000s respectively. $c^i$ and $\alpha^i$ are the coefficients to be estimated in the regression model with Gaussian noise $\varepsilon^i$. Marginal significance is observed in this model that indicates the potential usage of the information entropy to capture the market sentiment defined in this paper. The detailed results are presented in section 3.

## 3. Data and Numerical Results

The database used in the analysis is the Nikkei media marketing database for individual stock returns, index returns at the micro-second level transaction level in Japan.

The original database consists of 3944 firms, however, the sample was narrowed down to 416 firms by eliminating those firms that had more than 10 missing observations. This helped the convergence of the Kalman Filter estimation, which used the Nikkei 225 average as the market return. The sample period was for 2016

November (1083 observations), and the risk-free rate was obtained from the Bank of Japan 10–year bond yield. The risk premium is defined as the excess return over the risk-free rate in the analysis.

The high frequency tick data series data (intraday frequency) are converted into 5–minute VWAP (Volume Weighted Average Price) series, which are known to have a low level of micro-structure noise that is known to reduce the estimation accuracy. In addition, the first/last 5 minutes of data for morning and afternoon sessions are removed from the series to avoid any beginning-of-the-day/end-of-the-day price adjustment processes to be reflected in the analysis.

After all the adjustments were made, there were 1083 observations for each of the series. The Kalman Filter was applied on a daily frequency (57 observations per day) to obtain the long-term beta that is used to measure the systematic risk exposure to the market. This systematic risk exposure is compared to the average of the analyst forecasts (obtained from the QUICK database), which use the "end-of-month" forecasts that forecast the market level predicting the beginning of the following month. The analyst forecasts are used to represent the market sentiment level.

In Table 1, a single asterisk stands for a 10% significance level and a double asterisk stands for a significance level of 5%.

Observing the coefficients in Table 1, the signs of the coefficients all have positive signs indicating that the deviation from the analyst forecast level increases as the long-term beta increases. This sounds reasonable in the sense that the higher systematic risk induces noise in market that leads to an increase in long-term systematic risk. The negative relation implies the possibility that the larger the systematic risk, the smaller the deviation between the realized and analyst forecast level of the Nikkei225. The increase in long-term beta (proxy for the dynamically changing systematic risk) indicates an increase in systematic risk appears to have an impact on the market deviations from the analyst forecasts. In short, there might be some relation between the systematic risk and market sentiment defined in this paper.

Interestingly the coefficients for all of the firms considered, and those with the stock codes in the 8000s and 9000s are marginally significant at the 10% level, and the firms with the stock codes in the 5000s are statistically significant at the 5% level. Considering the rather small sample size, the results are rather appealing.

The next objective of this paper is to explore the possibility of using information

Table 1 Coefficients of estimated models dependent variable is the deviation between the systematic risk change and the analyst forecast level

| | coefficient | | Std. Error | t-stat. | Prob. |
|---|---|---|---|---|---|
| AVG_BE+1.6TAall | 0.160172 | * | 0.075988 | 2.107862 | 0.0502 |
| Constant | −0.151172 | ** | 0.067679 | −2.23366 | 0.0392 |
| AVG_BETA1000 | 0.008672 | | 0.033371 | 0.259857 | 0.7981 |
| Constant | −0.016965 | | 0.030567 | −0.55502 | 0.5861 |
| AVG_BETA2000 | 0.032002 | | 0.020715 | 1.544905 | 0.1408 |
| Constant | −0.040758 | * | 0.021679 | −1.88007 | 0.0773 |
| AVG_BETA3000 | −0.00404 | | 0.039407 | −0.10253 | 0.9195 |
| Constant | −0.005058 | | 0.042063 | −0.12024 | 0.9057 |
| AVG_BETA4000 | 0.038861 | | 0.031277 | 1.242484 | 0.2309 |
| Constant | −0.051113 | | 0.034504 | −1.48134 | 0.1568 |
| AVG_BETA5000 | 0.047626 | ** | 0.019135 | 2.488953 | 0.0235 |
| Constant | −0.059439 | ** | 0.021261 | −2.79568 | 0.0124 |
| AVG_BETA6000 | −0.011334 | | 0.054616 | −0.20753 | 0.8381 |
| Constant | 0.003323 | | 0.061307 | 0.054202 | 0.9574 |
| AVG_BETA7000 | 0.02 | | 0.033865 | 0.590593 | 0.5626 |
| Constant | −0.032965 | | 0.040843 | −0.80711 | 0.4308 |
| AVG_BETA8000 | 0.033572 | * | 0.018925 | 1.773913 | 0.094 |
| Constant | −0.043671 | * | 0.020704 | −2.10928 | 0.0501 |
| AVG_BETA9000 | 0.036946 | * | 0.019269 | 1.917345 | 0.0722 |
| Constant | −0.050769 | ** | 0.022796 | −2.22707 | 0.0397 |

entropy as an indicator to capture this market sentiment. The information entropy used in this paper is the Kullback-Leibler (1951) information criterion defined in section 2, and it should be noted that the relative entropy is derived from a uniform distribution that is often called the uninformative distribution. It should be noted that despite the fact that the distribution has the name "uninformative", it does not mean the distribution has no information. In fact, the uniform nature of the outcomes tells us that there is not a single price state of the world that may occur with a higher probability when compared to others. This leads to the motivation to set up the uniform assumption in the model because it serves as a conservative estimate assuming that one cannot forecast the future at all.

The results of the model that verifies the effectiveness of the information criterion implemented to identify the market sentiment level defined in this paper, are presented in the following Table 2, and the scatter plots are provided in the Appendix. There exists a marginal relationship overall.

In Table 2, single asterisk stands for a 10% significance level and a double asterisk stands for a significance level of 5%, a triple asterisk stands for a significance level at the 1% level.

Observing the coefficients in Table 2, the signs of the coefficients all have negative signs indicating that the information entropy level decreases as the long-term beta increases. This trend seems reasonable in the sense that the higher systematic risk reduces the future information of the price distribution. Interestingly the coefficients for all the firms, and those with the stock codes in

Table 2 Coefficients of estimated models Dependent variable Information Entropy

| | coefficient | | Std. Error | t-stat. | Prob. |
|---|---|---|---|---|---|
| AVG_BETAall | −0.435818 | ** | 0.204672 | −2.12935 | 0.0481 |
| Constant | −1.389476 | *** | 0.182292 | −7.62225 | 0 |
| AVG_BETA1000 | −0.047592 | | 0.089513 | −0.53168 | 0.6018 |
| Constant | −1.733414 | *** | 0.081991 | −21.1416 | 0 |
| AVG_BETA2000 | −0.06452 | | 0.057622 | −1.11971 | 0.2784 |
| Constant | −1.712083 | *** | 0.060304 | −28.391 | 0 |
| AVG_BETA3000 | −0.070546 | | 0.105014 | −0.67177 | 0.5108 |
| Constant | −1.701569 | *** | 0.112092 | −15.1801 | 0 |
| AVG_BETA4000 | −0.185427 | ** | 0.075841 | −2.44493 | 0.0257 |
| Constant | −1.575977 | *** | 0.083667 | −18.8364 | 0 |
| AVG_BETA5000 | −0.078719 | | 0.05723 | −1.3755 | 0.1868 |
| Constant | −1.692636 | *** | 0.063588 | −26.6189 | 0 |
| AVG_BETA6000 | 0.031193 | | 0.147413 | 0.211601 | 0.8349 |
| Constant | −1.810239 | *** | 0.165471 | −10.9399 | 0 |
| AVG_BETA7000 | 0.016149 | | 0.092257 | 0.175047 | 0.8631 |
| Constant | −1.794633 | *** | 0.111267 | −16.129 | 0 |
| AVG_BETA8000 | −0.064952 | | 0.053332 | −1.21788 | 0.2399 |
| Constant | −1.709007 | *** | 0.058345 | −29.2913 | 0 |
| AVG_BETA9000 | −0.137436 | *** | 0.046681 | −2.94416 | 0.0091 |
| Constant | −1.621233 | *** | 0.055225 | −29.357 | 0 |

the 4000s are marginally significant at the 5% level, where the firms with the stock codes in the 9000s are statistically significant at the 1% level. Considering the rather small sample size, the results appear to be promising.

## 5. Conclusion

This paper attempted to reveal the relation between the information level of the market, systematic risk, and market sentiment is investigated. The sentiment of the market is measured by the deviation between the analyst forecasts and the realization of the market, where the systematic risk is measured by the dynamically estimated the corporate beta using the Kalman Filter. There was a positive relation between the analyst forecast (proxy for sentiment) and corporate beta (proxy for systematic risk) at a marginally statistical level. In addition, the relation between the corporate beta (proxy for systematic risk) and entropy level of the market (proxy for market information) had a rather strong statistical relationship despite the small sample size examined.

The first achievement of the paper is that it has discovered that the analyst forecast deviations seem to be positively related to the dynamically estimated systematic risk measure (the long-term beta) using tick level data. This implies that market sentiment can be potentially captured via the dynamic estimation of systematic risk if the forecast deviation can be interpreted as market sentiment.

The second achievement of this paper is that it has revealed the potential usage of information entropy as a measure to scale the level of systematic risk measured by the long-term beta. This long-term beta is derived under the assumption that investors form their beliefs based on a Kalman filter, or in other words, that investors condition their expectations based on historical information. Thus, the information entropy measure appears to be a strong candidate to proxy the dynamically estimated systematic risk of the market.

This paper only has skimmed the surface of the information contained in the high frequency series (tick data) examined. There is future potential to investigate how potent the information is at a highly segmented level. However, the topic goes beyond the objective of this paper and is left for the future.

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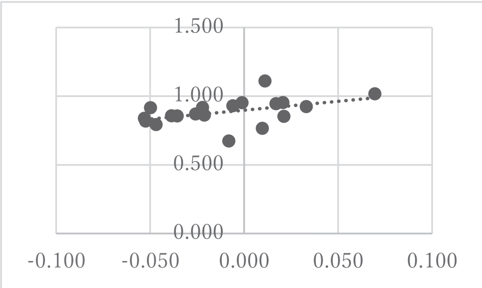
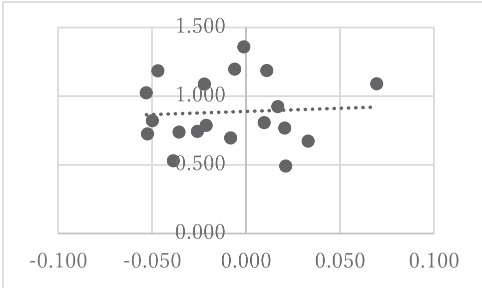
## Appendix

All Sectors (Beta v.s. Deviation)




Sector 1000's (Beta v.s. Deviation)

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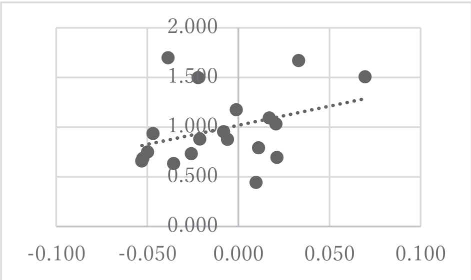


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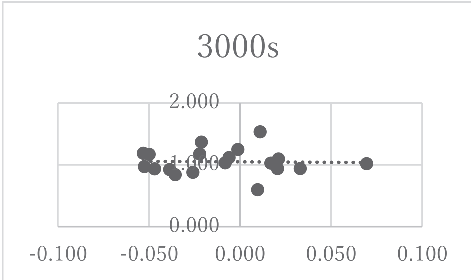


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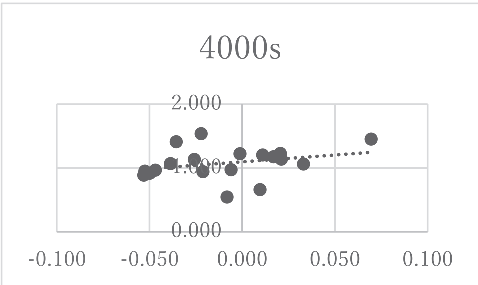


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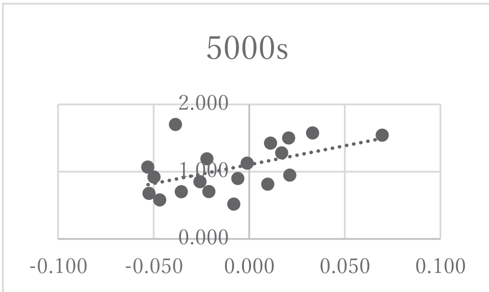


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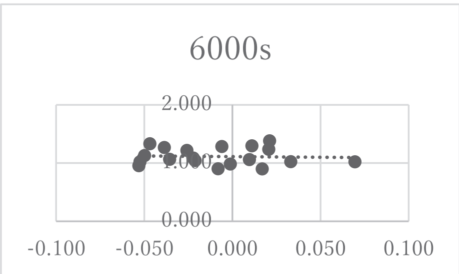


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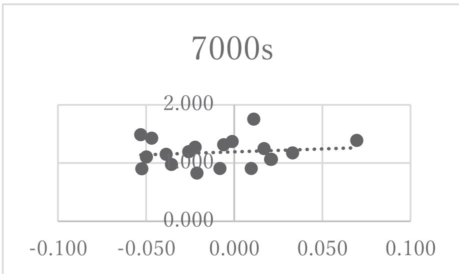

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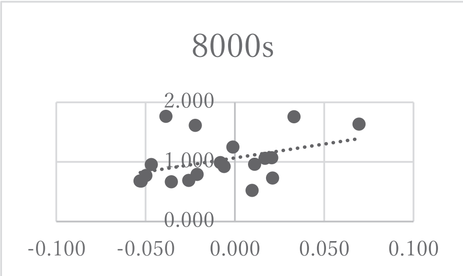
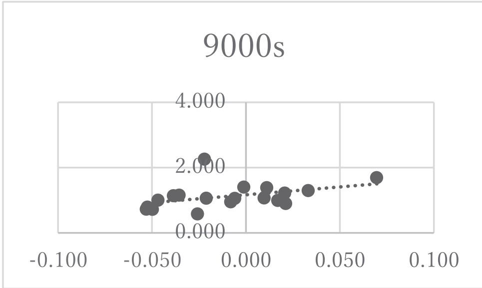
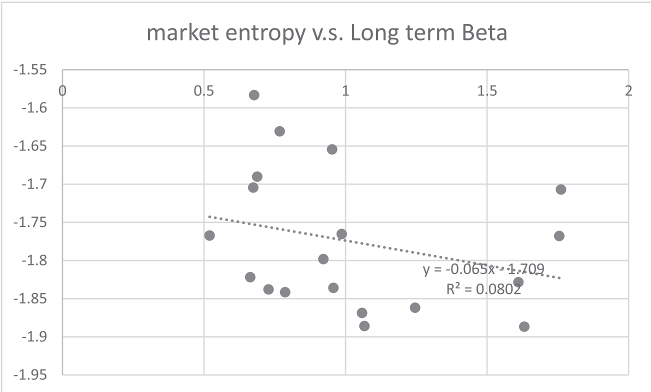



Sector 9000's (Beta v.s. Deviation)




All Sectors (Beta v.s. Market Information Entropy)

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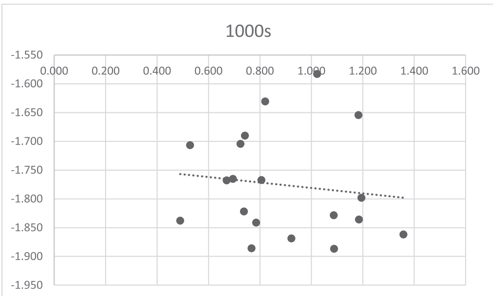
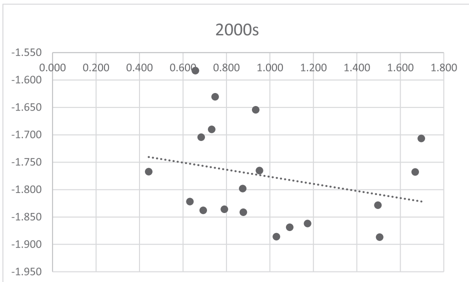



Sector 2000's (Beta v.s. Market Entropy)

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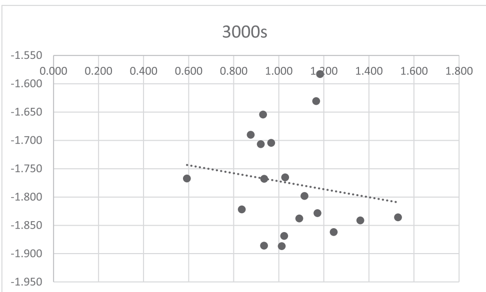


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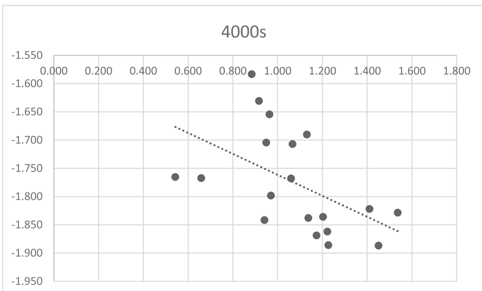

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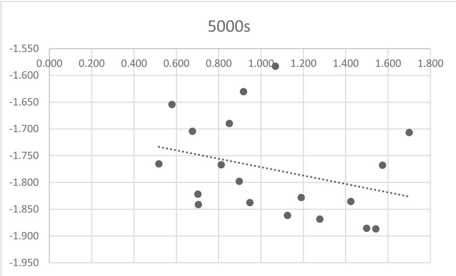
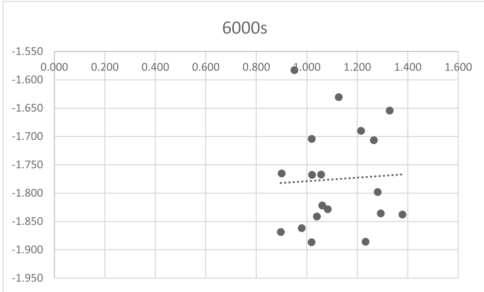



## Sector 6000's (Beta v.s. Market Entropy)

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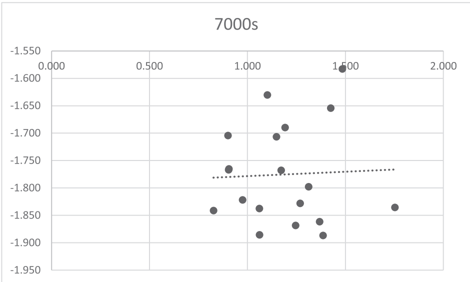
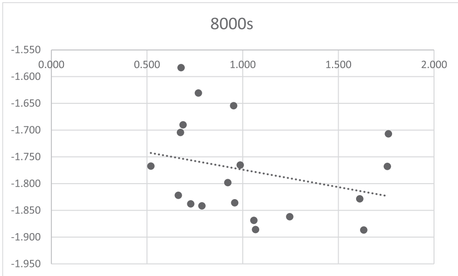



## Sector 8000's (Beta v.s. Market Entropy)




## Sector 9000's (Beta v.s. Market Entropy)